INTERACTION OF A TURBULENT BOUNDARY LAYER WITH AN ELASTIC PLATE

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An approximate method is presented for the determination of the efficiency with which an elastic plate reduces turbulent friction, and experimental results are also presented.

We know of a rather large quantity of experimental data needed as a basis for the mechanisms of interaction between an elastic plate and a turbulent flow, and for the development of appropriate theories. The most familiar models of such mechanisms are given in [1] (absorption by an elastic plate of the energy of turbulent pulsations), [2] (interferential frequency interaction of the vibrations of an elastic plate with the vibrations of a viscous substrate), [3] (the frequency interaction of elastic-plate vibrations with the ejecta from a viscous substrate), [4, 5] (the structural interaction of an elastic plate with the perturbing motion of the boundary layer).

The basis for all of these hypotheses, in part or totally, is the idea of resonance frequency and energy conformity in the vibrations of an elastic plate to pressure pulses and the displacement of a boundary layer. In this connection, of decisive importance is a study of the quantitative relationships governing the development of forced and intrinsic vibrations of the elastic plate, caused by pulsations of the boundary layer. The difference in these hypotheses lies in the fact that the induced force is treated either in the form of a harmonic vibration (monoharmonics, a static field of harmonic vibrations) or in the form of discrete bursts of a pulsation field (for example, determined by the frequency of pulses from a viscous substrate).

Therefore, for the formulation and solution of the corresponding problem it is necessary to specify the boundary conditions at the wall, for example, in the form of harmonic vibrations which account for the mechanical characteristics of an elastic plate. Differential equations of liquid motion are given in the form of Prandtl equations and continuity equations, or enlarged by the equation of boundary-layer energy balance. However, the theoretical problem is complicated by the fact that the boundary conditions depend on the solution of a system of equations, i.e., the problem remains indeterminate. To remove this indeterminacy, we introduce a variety of assumptions to link the vibrations at the wall or in the boundary layer, for example, either in terms of a phase shift, or in terms of conductivity, as is done in acoustics or electrodynamics.

Thus to solve the problem we have to study the quantitative relationships governing the vibrations of an elastic plate subjected to isolated, periodic, or statistical interrelated external disturbances. The nature of this interrelationship is governed by the construction and material composition of the elastic plate and, in first approximation, can be studied on the basis of familiar methods from the theory of elasticity and the damping of vibrations. Elastic plates in the form of membrane surfaces are more conveniently studied by means of the Voigt–Kelvin model. The equations of motion for such multilayered plates, used for purposes of calculating hydrodynamic stability [4, 6], are written in the form [7]:

$$(T_1+T_2)\frac{\partial^2\varepsilon_y}{\partial x^2}-(M_1+M_2)\frac{\partial^2\varepsilon_y}{\partial t^2}-\eta_t\frac{\partial\varepsilon_y}{\partial t}-\\-(E_1'+E_2'+E_3')\varepsilon_y=P-\sigma_{\text{bend}}. \tag{1}$$

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An analogous Kornekki equation was used in [6], and similar equations are also to be found in [3].

If the elastic plate is fashioned out of a monolithic single-layer or multilayer material, then Eq. (1) becomes considerably more complex. An equation of motion was derived in [1] for a single-layer monolithic elastic plate; this equation defines the relationship of the strain tensor to the stress tensor:

$$\rho \frac{\partial^2 \xi_j}{\partial t^2} = (G+\lambda) \frac{\partial}{\partial x_j}[\Delta] + G\nabla^2 \xi_j + \frac{1}{3} G\tau_k \frac{\partial}{\partial x_j}[\dot{\Delta}] + G\tau_k \nabla^2 \dot{\xi}_j. \tag{2}$$

When the elastic plate interacts with a turbulent boundary layer, we find a constant statistically distributed excitation of vibrations within the plate. In accordance with the well-known mechanical properties of elastic high-molecular materials, the energy of the boundary layer acting on the plate is partially accumulated ($\tilde{E}$) in the elastomer and partially dissipated (Φ). This latter quantity can be calculated by introducing an absorption factor $r_1 = \tau\Phi/\tilde{E}$ for a coating segment of unit width, of thickness h_i and length $\ell = 2\pi/\alpha$ over a period of vibration relative to the initially accumulated energy. That part of the energy accumulated in the coating can be determined by introducing the coefficient $r_2 = (\tilde{E} - \tau\Phi)/\tilde{E}$.

The expressions for $\tilde{E}$ and Φ have the form [1]

$$\tilde{E} = \iint\limits_{(s)} \left\{ G\left[\left(\frac{\partial \xi_x}{\partial x}\right)^2 + \left(\frac{\partial \xi_y}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial \xi_x}{\partial y} + \frac{\partial \xi_y}{\partial x}\right)^2 \right] + \right.$$
$$\left. + \frac{1}{2}(K - 2/3G)\left(\frac{\partial \xi_x}{\partial x} + \frac{\partial \xi_y}{\partial y}\right)^2 \right\} ds, \tag{3}$$

$$\Phi = 2G\tau_k \iint\limits_{(s)} \left\{ \frac{2}{3}\left[\left(\frac{\partial \dot{\xi}_x}{\partial x}\right)^2 - \frac{\partial \dot{\xi}_x}{\partial x}\frac{\partial \dot{\xi}_y}{\partial y} + \left(\frac{\partial \dot{\xi}_y}{\partial y}\right)^2 \right] + \right.$$
$$\left. + \frac{1}{2}\left[\frac{\partial \dot{\xi}_x}{\partial y} + \frac{\partial \dot{\xi}_y}{\partial x}\right]^2 \right\} ds. \tag{4}$$

Calculations [8] and experiments [9] showed that as long as the thickness of the elastic layer does not exceed the minimum wavelength of the wavelength range under consideration, a periodicity is observed in the variations in density, in the magnitude of the accumulated energy, and in the rate of dissipation.

As the thickness of the layer increases, there is a redistribution in the densities of Φ and $\tilde{E}$ and their maxima shift toward the region of greater wavelengths. Because of an increase in viscosity, the maximum values of the accumulated energy are shifted into the low-frequency band. Within the covered range of parameters for the mechanical properties of the elastomer [$K = 1 \cdot 10^8$ N/m^2, $G = 2 \cdot 10^5$ N/m^2, $\eta = 5 \cdot (10^3 - 10^4)$ N·sec/m^2, $h = 5 \cdot (10^{-3} - 10^{-2})$ m] energy is accumulated and dissipated in the range $f = 0$-500 sec^{-1}.

Thus, by varying the number of layers, their construction and material properties, we can achieve a variety of absorption effects with regard to the pulsation energy of the boundary layer in various energy-carrying boundary-layer frequency bands.

However, we know that in fabricating elastomer materials it is difficult to maintain specific mechanical properties in an adequate number of the fabricated plates.

Moreover, the mechanical parameters depend strongly on the temperature, while the complex modulus of elasticity is also dependent on the frequency of the forced vibrations. Therefore, for successful application of elastomers in actual practice it is necessary to develop an approximate theory for the selection of elastomers with a sufficiently high probability of ensuring the specified parameters of their mechanical characteristics. Essentially this method involves the following. An elastic plate interacts with a flow when the frequency bands are approximately coincident and correspond to the regions of greatest energy-carrying frequencies in a turbulent boundary layer, as well as to the greatest absorption of mechanical vibrations by the elastic surface. In this case, the mechanical characteristics of the elastomer must be such that the energy of the turbulent pressure and shear pulsations is adequate to excite surface vibrations in the elastomer. A specific phase shift must be achieved between the forced vibrations and those induced within the elastomer [10], and the

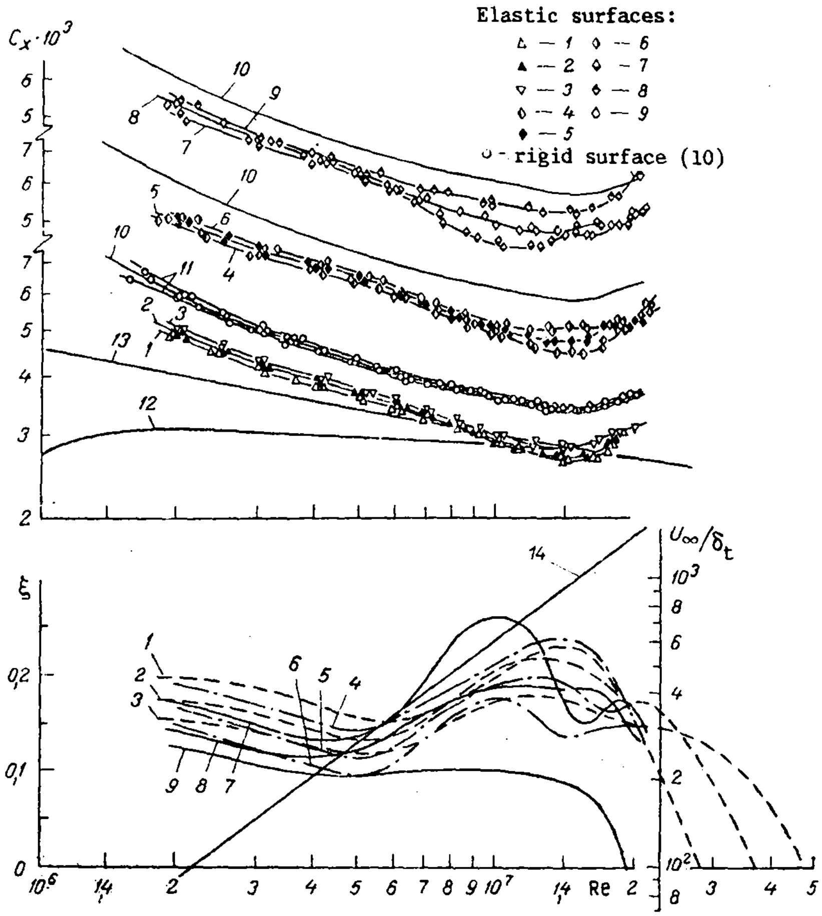

Fig. 1. Resistance coefficient C_x of longitudinally streamlined elastic (1-9) and rigid (10) cylinders, as well as the coefficient ξ of resistance reduction, as functions of the Reynolds number (the number of the elastic surface corresponds to the number in Table 1); 11) mean square error in measurement of $\sigma_{\Delta C_x}$ (Re); 12, 13) plate resistance in transition and turbulent flows; 14) range of energy transporting frequencies U_∞/δ_t of the pulsation load in the boundary layer as a function of Re_L at the end of the elastic insert. U_∞/δ_t, sec^{-1}.

TABLE 1. Geometric Parameters of Elastic Plates

No.	Elastomer	h/L	No.	Elastomer	h/L
1	PPU-3-A	0,01062	6	PPU-3-B	0,00265
2	PPU-3-A	0,00531	7	PPU-3-C	0,01062
3	PPU-3-A	0,00265	8	PPU-3-C	0,00531
4	PPU-3-B	0,01062	9	PPU-3-C	0,00265
5	PPU-3-B	0.00531			

vibrations themselves should not exceed the limits of allowable magnitude [2, 3, 11]. In this case, the elastomer will take the energy of the turbulent boundary layer and partially absorb it, which results in a thickening of the viscous substrate, in a reduction in friction at the wall, and in an increase in the period of the pulses from the viscous substrate.

The integral $2\int_{\omega_1}^{\omega_2} \mathrm{tg}\,\varphi(\omega)d\omega$ defines the area encompassed by the curve of the function relating the tangent tan φ of the loss angle to the frequency ω of the forced vibrations. Usually, the loss curve exhibits a single vibration-frequency maximum at which we find the greatest loss of energy in the mechanical vibrations of the material, i.e., the elastomer scatters the greatest amount of energy in that frequency band. Therefore, an elastic plate will be effective when the indicated frequency band coincides with the energy-carrying spectrum of turbulent pulsations for the corresponding range of flow velocities.

Based on the above, the choice of a specific elastic material begins with the experimental determination of the curve $\tan\varphi(\omega)$. We know that the frequency U_∞/δ_t corresponds for each Re to the range of the highest energy-carrying pulsations. Thus, at the frequency U_∞/δ_t, which corresponds to $\tan\varphi_{max}$, for the given elastic plate we must establish the greatest effect of reducing frictional drag.

Results from an experimental verification of the proposed method are given in [11] for elastic inserts mounted in the form of rectangular panels in a hydrodynamic test stand and on a tow plate. Despite the verification, the accuracy of the measurements raised some doubts, since the data obtained during the tests on the plates depend significantly on the longitudinal pressure distribution, as well as on the experimental technique. In this connection, the studies were performed with tensiometric inserts in the form of longitudinally streamlined cylinders. The test method is described in [12] in which it is demonstrated that such a tensiometer simulates the measurements on a plate. The frictional resistance of the elastic cylinders was determined by towing them through the channel. A hollow metal cylinder with a polished surface served as the referenced standard of a rigid surface. The outside diameter of the elastic cylinders, as well as the roughness, correspond to this standard.

The data on certain mechanical parameters of the elastomers employed here are presented in [11]. Their dynamic properties were determined by the method of forced nonresonant vibrations on a specially developed experimental installation. The dynamic modulus of elasticity for a number of elastic materials is significantly greater than for the materials tested earlier [11]. In our experiments the rigidity of the elastic materials varied not only as a consequence of the difference in the quantity E', but also because of changes in their geometric parameters (see Table 1).

Results from the measurement of the coefficient of friction for longitudinally streamlined elastic cylinders are presented in Fig. 1. The frequencies U_∞/δ_t were calculated from the towing velocity and from the Re_L number, and the quantity $\delta_t(x)$ was determined from formula $\delta_t(x) = 0.37x Re_x^{-0.2}$ where $x = L$.

We observed the hydrodynamic effect on all of the elastic plates over the entire range of Re numbers. The exception (specimen No. 9) was a result of the slight separation of the elastic wall from the base at the beginning of the cylinder, which in the case of Re_{max} resulted in an increase in the resistance relative to that of the standard.

In all cases, the maximum effect is determined by correlation of the dynamic elastomer properties with the energy-carrying frequencies of the pulsation load for the turbulent boundary layer. We will use specimen No. 4 to examine the results shown in Fig. 1. Below $Re = 5\cdot10^6$ the effect is almost constant and a result of the fact that according to curve 14 (see Fig. 1), the frequency of the pulsation load does not exceed ~250 sec^{-1}. According to the data of Fig. 2 in [11], this material in the indicated frequency range exhibits a constant $\tan\varphi \approx 0.6$. With a subsequent increase in the Re numbers there is a rise in the frequency of the load, and according to [11], $\tan\varphi = 0.9 = \max$ when $\omega \approx 500\text{-}800\ sec^{-1}$. According to curve 14, this corresponds to $Re \approx 1.3\cdot10^7$. Consequently, the established maximum effect corresponds precisely to those frequencies at which the elastomer exhibits maximum absorption properties ($\tan\varphi = \max$). This also serves to explain that according to Table 1, for specimens Nos. 4-6, $h_4 > h_5 > h_6$. In this case $(\xi_{max})_4 > (\xi_{max})_5 > (\xi_{max})_6$, which is determined by the rigidity of the material and, consequently, by the dynamic properties and the frequency shift ($\tan\varphi_{max}$).

NOTATION

x and y, longitudinal and vertical coordinates, m; ξ_j, shift along the coordinate axes, m; j = 1, 2, the x and y coordinates; ε_y, shift in the direction of the pressure effect, m; $\ell = 2\pi/\alpha$, unit length of the plate; α, wave number; h_i, thickness of the elastic layer, m; $\Delta = (\partial\xi_x/\partial x) + (\partial\xi_y/\partial y)$, strain tensor; L, distance from the beginning of the tensiometer to the end of the tensiometric insert, m; δ_t, thickness of the turbulent boundary layer, m; t, time, sec; τ_k, strain relaxation time, sec; f, frequency of boundary-layer vibration, Hz; ω, angular frequency of the forced vibrations, sec^{-1}; U_∞/δ_t, maximum energy-carrying boundary-layer frequency, sec^{-1}; U_∞, velocity, m/sec; ∇^2, Laplace operator; Re, Reynolds number; T_i, tension within the elastic layer, N/m^2; M_i, vibrating mass of the elastomer, $kg \cdot sec^2/m^3$; ρ, density of the elastomer, kg/m^3; E_i, modulus of elasticity, N/m^2; $E_i' = E_i/h_i$, rigidity of the elastic layer, N/m^3; i, number of plate layer; $E = E' + E''$, complex modulus of elasticity; $E'(\omega)$, modulus of elasticity; $E''(\omega)$, loss modulus; $\tan\varphi = E''/E'$, energy-loss coefficient (the loss tangent); P, surface pulsation pressure, N/m^2; σ_{bend}, bending stress in the plate, N/m^2; G, shear modulus, N/m^2; η_i, viscosity of the elastic layer, $N \cdot sec/m^2$; λ, Lamé parameter; $K = \lambda + {}^2/_3 G$, volumetric modulus of elasticity, N/m^2; $\eta_k = G\tau_k$, shear viscosity, $N \cdot sec/m^2$; $\tilde{E}$, portion of the boundary-layer energy accumulated in the elastomer; Φ, dissipated portion of the energy; $r_1 = \tau\Phi/\tilde{E}$, coefficient of absorption for potential energy; $r_2 = (\tilde{E} - \tau\Phi)/\tilde{E}$, coefficient of the potential energy of the boundary layer accumulated in the elastomer; C_x, coefficient of frictional resistance; $\xi = (C_{x_{rigid}} - C_{x_{elast}}) C_{x_{rigid}}^{-1}$, coefficient of reduction in frictional resistance.

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INVESTIGATION OF EHD FLOW BASED ON A NUMERICAL SOLUTION OF THE NAVIER–STOKES EQUATIONS

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The proposed numerical method is used to examine the physical pattern of EHD flow at a high-voltage flat electrode.

Electrohydrodynamic (EHD) flows of low-conductivity liquids from a high-voltage electrode have been studied both theoretically and experimentally in numerous works (see, for example, [1-4]). Despite a superficial similarity in the EHD flow patterns and those of a Landau–Slezkin "submerged jet," the latter cannot be treated as a sufficiently exact model of EHD flow. In particular, in the case of an immersed electrode it makes no provision for the effect of the friction of the jet against the wall of the vessel. An estimate is presented in [1] of the possible velocity of isothermal electrical convection as part of a study of the laminar flow of an incompressible dielectric liquid around an electrode. According to [1, 2] the flow is caused primarily by a Coulomb force acting on the space charge that is formed because of a nonuniformity in electrical conductivity that is weak but different from zero. A semiempirical formula was proposed in [1] for a steady-state charge, and by means of this formula it became possible qualitatively to describe the phenomena of electrical con-

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